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The Temperature Dependence of the $SU(N_{\mathrm{c}})$ Gluon Condensate from Lattice Gauge Theory

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Abstract
An analysis of the temperature dependence of the leading contributions to the gluon condensate for $SU(N_{\mathrm{c}})$ lattice gauge theory is presented using the data from recent Monte Carlo simulations. The gluon condensate is calculated directly from the new lattice calculations of the interaction measure. It is shown how these computations provide a simple picture for the melting of the condensate around the deconfinement temperature, and the fact that it is negative, and increases in magnitude, above T_{c}. We close with a discussion of the implications for full QCD of recent results from simulations including fermions.

1 Introduction

We discuss here the consequences for the gluon condensate at finite temperature of the recent high precision lattice results for the equation of state in $SU(2)$ [1] and $SU(3)$ [2], and compare with various other calculations of the expected high temperature behavior of the condensate. The relationship between the gluon condensate and equation of state arises due to the scale variance of quantum chromodynamics (QCD), the trace anomaly, which relates the trace of the energy momentum tensor to the square of the gluon field strenth through the renormalization group beta function. We expand on the work presented in [3], where the consequences of the new finite temperature lattice data for $SU(N_\mathrm{c})$ gauge theory for the gluon condensate were discussed.

These ideas have been studied for finite temperature by Leutwyler [4] in relation to the problems of deconfinement and chiral symmetry. He discusses in detail the relationship between the trace anomaly and the gluon condensate, based on the interaction between Goldstone bosons in chiral perturbation theory. The condensate has also been investigated directly on the lattice using plaquette operators by [5,6].

The energy momentum tensor at finite temperature $T^{\mu\nu}(T)$ can be separated into the zero temperature part, $T_0^{\mu\nu}$, and the finite temperature contribution $\theta^{\mu\nu}(T)$:

$$T^{\mu\nu}(T) = T_0^{\mu\nu} + \theta^{\mu\nu}(T). \tag{1.1}$$

The zero temperature part, $T_0^{\mu\nu}$, has the standard problems with infinities of any ground state, and so is not readily calculable on the lattice. The finite temperature part, which is zero at zero temperature, is free of such problems, and the diagonal elements of $\theta^{\mu\nu}(T)$ are calculated in a straightforward way on the lattice. The trace $\theta_\mu^\mu(T)$ is connected to the thermodynamic contribution to the energy density ϵ and pressure P for relativistic fields and relativistic hydrodynamics [7]:

$$\theta_\mu^\mu(T) = \epsilon - 3P. \tag{1.2}$$

There are no other contributions to the trace for QCD on the lattice. The heat conductivity is zero, as there is no non-zero conserved quantum number, and there is no velocity gradient in the lattice study, hence no contribution from viscosity terms.

The dimensionless interaction measure $\Delta(T) \times T^4$ is equal to the thermal ensemble expectation value of $(\epsilon - 3P)/T^4$. So from equation (1.2) above the interaction measure and the expectation value of the temperature dependent part of the energy momentum tensor are linked by

$$\Delta(T) \times T^4 = \theta_\mu^\mu(T). \tag{1.3}$$

An $SU(N_\mathrm{c})$ calculation of $\Delta(T) \times T^4$ on a lattice of size $N_\sigma^3 \times N_\tau$ with lattice spacing a proceeds as follows (see [1,2] for further details). From the action expectation value

at zero temperature, P_0, as well as the spatial and temporal action expectation values at finite temperature, P_σ and P_τ respectively and N_τ the number of temporal steps, the dimensionless interaction measure $\Delta(T)$ [8] is given by:

$$\Delta(T) = -6N_c N_\tau^4 a \frac{dg^{-2}}{da} \left[2P_0 - (P_\sigma + P_\tau) \right].$$ (1.4)

The crucial part of these recent calculations is the use of the full lattice beta function, $\beta_{\text{fn}} = adg^{-2}/da$ to obtain the scale a from the bare coupling g^2 [8, 9]. The physical value of a is then fixed via the string tension. Without this accurate information on the temperature scale in lattice units it would not be possible to make any claims about the behavior of the gluon condensate[a].

Let $G_a^{\mu\nu}$, where a is the color index for $SU(N_c)$, denote the gluon field strength tensor. The quantity G^2 is then defined [4], using the beta function $\beta_{\text{fn}}(g)$ with g the bare coupling, to be

$$G^2 = \frac{-\beta(g)}{2g^3} G_a^{\mu\nu} G_{\mu\nu}^a.$$ (1.5)

For a scale invariant system, such as a gas of free massless particles, the trace of the energy momentum tensor, equation (1.3), is zero. A system that is scale variant, perhaps from a particle mass, has a finite trace, with the value of the trace measuring the magnitude of scale breaking. At zero temperature it has been well understood from Shifman et al. [10] how in the QCD vacuum the trace of the energy momentum tensor relates to the gluon field strength squared, $\langle G^2 \rangle_0$. A finite temperature gluon condensate $\langle G^2 \rangle_T$, related to the degree of scale breaking at all temperatures, can be defined to be equal to the trace. As the scale breaking in QCD occurs explicitly at all orders in a loop expansion, the thermal average of the trace of the energy momentum tensor, and hence the gluon condensate, need not go to zero above the deconfinement transition. Using the temperature dependent part of the trace, and the value of the condensate at zero temperature, the finite temperature gluon condensate can be written

$$\langle G^2 \rangle_T = \langle G^2 \rangle_0 - \theta_\mu^\mu(T),$$ (1.6)

where the brackets with the subscript T means thermal average.

In the next two sections we evaluate the gluon condensate as defined in equation (1.6) above using the lattice data on $\theta_\mu^\mu(T)$ in $SU(2)$ and $SU(3)$ gauge theories. This is followed by simple melting model for the gluon condensate in pure gauge theories. After this we look into the high temperature behavior of $\Delta(T)$ in order to determine the reliability of its thermodynamical properties in relation to some earlier models. We close the body of the paper with a discussion of the gluon condensate in full QCD, where the fermions may change the behavior. Finally we conclude with an evaluation of the present situation regarding the properties of condensates in the presence of strong interactions.

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[a]The temperature scale in physical units will have an error due to the error on the experimental value of the string tension used to connect the lattice units to physical units.

Table 1: The lattice results for SU(2) from [1] and for SU(3) from [2] for the interaction measure in the continuum limit of pure gauge theory. Both the value at the phase transition and at the peak are given.

		$T(\text{GeV})$	$\Delta(T)$	$\Delta(T) \times T^4$
$SU(2)$	T_c	0.290	0.381	0.00269
	T_{peak}	0.341	1.059	0.0143
$SU(3)$	T_c	0.264	1.145	0.00556
	T_{peak}	0.290	2.266	0.0197

2 $SU(2)$ pure gauge theory

In this section we present our evaluations for the gluon condensate for the $SU(2)$ lattice gauge theory. Taking the published data [1] for $\Delta(T)$, and using equations (1.6) and (1.3) we obtain the gluon condensate $\langle G^2 \rangle_T$. Since the lattice is not yet able to unambiguously obtain the condensate at zero temperature, but only calculates the contribution due to finite temperature, we need an additional input for $\langle G^2 \rangle_0$ to set the scale at zero temperature. Although there is some numerical uncertainty in the exact value of $\langle G^2 \rangle_0$, it has no direct influence on the high temperature properties of the gluon condensate [4].
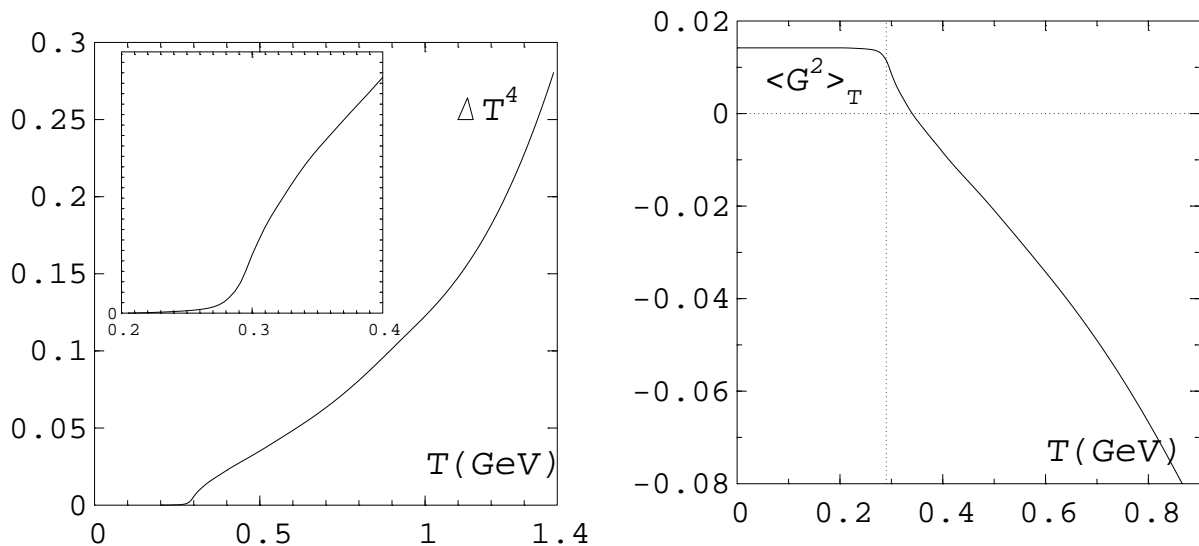

Figure 1: Figure (a) shows $\Delta(T) \times T^4$ for $SU(2)$ gauge theory in terms of $(\text{GeV})^4$. The insert shows a magnified region around the phase transition. Figure (b) shows the corresponding condensate. The vertical dotted line indicates the critical temperature $T_c = 0.290$ GeV.

From the lattice data for $SU(2)$ [1] we have defined the value for $\langle G^2 \rangle_0$ from the peak of $\Delta(T)$ to be 0.0143GeV^4, which is consistent with the range of values (see the end of section 4 taken from the previous $SU(2)$ computations [11–14]. The values corresponding to T_c and T_{peak} for $\Delta(T)$ and $\Delta \times T^4$ are tabulated in table 1. Our main reason for this choice for the vacuum value is that the peak of $\Delta(T)$ represents the point of fastest change of the thermal properties of the gluon condensate after which the growth of $\Delta(T) \times T^4$ becomes somewhat slower. Using these values we show in Figure 1(a) the behavior of $\theta^\mu_\mu(T)$ over various temperature ranges. Figure 1(b) shows the corresponding behavior of the gluon condensate over a smaller temperature range. We can clearly see that the gluon condensate goes very rapidly negative for temperatures not very far above T_c. note that within the range of numerical values usually considered for $\langle G^2 \rangle_0$ the temperature at which $\langle G^2 \rangle_T$ goes through zero changes little, and remains above T_c for all these values. Furthermore, varying $\langle G^2 \rangle_0$ in this range does not cause any qualitative change in the high temperature behavior of $\langle G^2 \rangle_T$.

3 $SU(3)$ pure gauge theory

Analogous to our approach in the previous section we use the SU(3) data of [2] in order to compute $\Delta(T) \times T^4$. After having done this computation we are able to find $\theta^\mu_\mu(T)$ and thus obtain $\langle G^2 \rangle_T$ from equation (1.6). Again we have taken the value of $\Delta(T) \times T^4$ from the peak of Δ to define where $\langle G^2 \rangle_T$ goes to zero. This leads to a value of $0.0197(\text{GeV})^4$ for $\langle G^2 \rangle_0$, which is somewhat larger than the value of $0.012(\text{GeV})^4$ that is usually extracted through the sum rules from charmonium decay. Nevertheless, our value is still within the range of plausible values [15]. Furthermore, we should note that for pure gauge theory one expects a gluon condensate that is larger than in full QCD.

The values of $\Delta(T) \times T^4$ are given in table 1. The plots in Figure 2 show more rapid changes for $SU(3)$ than the corresponding plot in Figure 1 for $SU(2)$. The qualitative structure is the same though for both SU(2) and SU(3), both having $\langle G^2 \rangle_T$ dropping rapidly around T_c and a continuation into negative values at higher temperatures. These results, from a first principles QCD calculation, are the same as the chiral perturbation theory analysis previously made by Leutwyler [4].

The results presented in these two sections differ considerably from earlier direct calculations [5, 6]. This seems to be due to our using a definition that does not require a technically difficult seperation of the condensate from a perturbative contribution. Also crucial is the use of a fully non-perturbative lattice calculation of the interaction measure, including the full non-perturbative beta function.

However, care should be taken in drawing conclusions about the physical consequences of these results, as the condensate at finite temperature need not reflect the same physics as at zero temperature. For example, as we will discuss below, there are additional scale breaking thermal contributions to the condensate which do not arise from confinement.

4   Simple Melting Model for the Gluon Condensate

The results of the previous two sections provide the temperature dependence of the condensation of gluons for $SU(2)$ and $SU(3)$ lattice gauge theories. There we defined the condensate to be related to the scale breaking at all temperatures, that is the amount by which $\epsilon - 3P$ differs from zero. The value at $T = 0$ is used to set the scale. Other approaches, such as a direct calculation based on plaquette correlators [5, 6, 14], will be related to this one by renormalisation constants, either additive or multiplicative, or both.

In Figures 1(b) and 2(b) we have shown $\langle G^2 \rangle_T$ for $SU(2)$ and $SU(3)$ pure gauge theory respectively. In both cases the general form of this finite temperature condensate is very much the same. At temperatures well below T_c the vacuum condensate $\langle G^2 \rangle_0$ dominates. From slightly below to just above T_c there is a sharp drop in the amount of the gluon condensate present, and a rapid approach to zero. For temperatures above the temperature where $\langle G^2 \rangle_T$ has reached zero, we see that $\Delta(T) \times T^4$ is continually growing in absolute value, which causes $\langle G^2 \rangle_T$ to become increasingly negative.

Thus the processes at lower temperature responsible for 'pulling' gluons out of the condensate continue at higher temperature, driving the condensate further down to negative values. There are known to be residual interactions in the high temperature phase, particularly in the spatial (magnetic) direction [2, 16]. This leads to a picture of the condensate at finite temperatures in which the condensate, with zero total momentum,
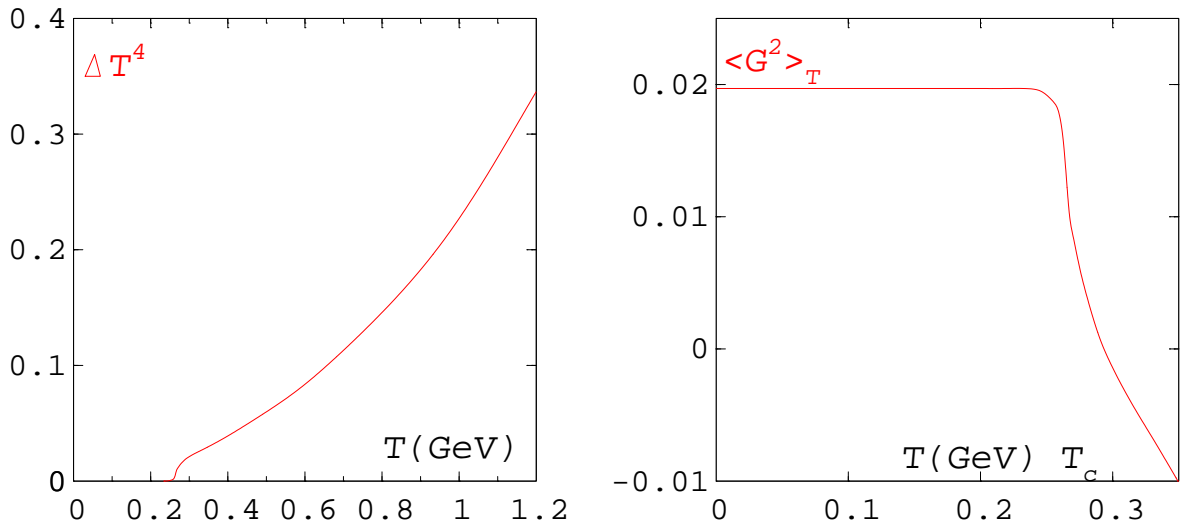

Figure 2: Figure (a) shows $\epsilon - 3P$ for $SU(3)$ gauge theory, extrapolated to the continuum limit. Figure (b) shows the corresponding condensate. The critical temperature is $T_\mathrm{c} = 0.264$ GeV.

is composed of thermal gluon pairs in which one gluon has momentum p, the other $-p$. Since the typical momentum is $p \sim T$, higher temperatures correspond to both higher momentum states, and a higher density of states, contributing.

Here we shall discuss an oversimplified model with boson fields $B(p)$, and define the quantity $\langle B^2 \rangle_T$, in analogy to the gluon condensate at finite temperatures. Here we shall suppose that the oppositely directed momenta of the two fields are correlated by an interaction in momentum space $V(|p_1 - p_2|)$, or equivalently, since we have $p_1 = -p_2 = p$, $V(2p)$.

Thus the effect of finite temperature in this model just leads to an expression of the following form: $\langle B(-p)V(2p)B(+p)\rangle_T$, where the function $V(2p)$ could be any function, of less than exponential order, of the magnitude of the momentum. This could be a positive power of $|p|$, or an algebraic function expandable in positive powers of p. A modified Bessel function $K_\nu(2p)$ fulfills all the requirements. From this we may write the equation for the process relating to the vacuum value $\langle B^2 \rangle_0$ as follows:

$$\langle B^2 \rangle_T = \langle B^2 \rangle_0 + \langle B(-p)V(2p)B(+p)\rangle_T. \tag{4.1}$$

In order to evaluate the last term in this equation, we expand $V(2p)$ in powers of the argument $2p$ and approximate the field distributions with a simple Planck distribution function at finite temperatures $n(+p)$ and $n(-p)$. The integral then takes the form

$$\langle B(-p)V(2p)B(+p)\rangle_T = \int \frac{d^3p}{(2\pi)^3} \, n(-p)V(2p)n(+p), \tag{4.2}$$

where the spatial volume has been divided out leaving a density term. After carrying out the expansion in powers of the momentum indicated above, and using the spherical symmetry in momentum space, we may exactly evaluate this integral to obtain

$$\langle B(-p)V(2p)B(+p)\rangle_T = \frac{-1}{2\pi^2} \sum_{j=0}^{\infty} V_j 2^j T^{3+j}(j+2)!\zeta(j+2), \tag{4.3}$$

where $\zeta(s)$ is the Riemann zeta function with the argument s. We now can carry out the numerical evaluation of the above terms, which yields

$$\langle B(-p)V(2p)B(+p)\rangle_T = -0.16667V_0T^3 - 0.73076V_1T^4 - ..., \tag{4.4}$$

where the V_j are the coefficients of the expansion of the interaction. As an example of these results, let us suppose that the only nonzero term in the expansion is V_0, which is just a constant. Then the condensation equation becomes

$$\langle B^2 \rangle_T = \langle B^2 \rangle_0 - 0.16667V_0T^3. \tag{4.5}$$

This equation is analogous to the Bose-Einstein condensation (BEC) equation, with the condensation temperature defined as the cube root of $6\langle B^2 \rangle_0/V_0$. However, in contrast to

the usual BEC in statistical particle systems, equation (4.5) applies both above and below the condensation temperature. Therefore, $\langle B^2 \rangle_T$ can become negative at sufficiently high temperatures for a system which has only boson fields present.

A reasonable picture of this process seems to be that the attractive interactions of the gluons at these very high temperatures causes a sort of 'antiscreening' effect, a continuation of the melting process to values exceeding that of the vacuum condensate. Before we are able to continue this discussion in more detail, we must consider some points in the analysis of the vacuum contributions to the gluon condensate. The classical analysis of the gluon contributions at the lowest order in the operator product expansion [10] arrives at a value for the $\langle G^2 \rangle_0$ term of about 0.012GeV^4. Although the numerical confirmation of this analysis has later shown some difficulties due to certain singularities, the basic method of the operator product expansion for obtaining hadron properties from the QCD sum rules is well confirmed [17].

A very recent study [15] shows a rather large range of values of $\langle G^2 \rangle_0$ between 0.0127 and 0.0355 GeV^4. Also a number of computations for the pure lattice gauge theories have been carried out for $SU(2)$ [11–13] as well as for $SU(3)$ [14]. A check of the range of G^2 for $SU(2)$ using the relationship of Λ_L to T_c for the new computations on $SU(2)$ [1] with T_c at 0.290 GeV gives values around 0.0059 to 0.0236 GeV^4. These values , although somewhat smaller than the QCD values, are still quite reasonable. The numbers from lattice computations for $SU(3)$ [14] yield values much too large when the new computations for $SU(3)$ [2] relating T_c to Λ_L are used. In this case we choose the values from QCD. The discussion above leading to a melting of the gluon condensate and a continuation of the same process into the high temperature region brings in the question of the thermodynamical properties of such a system. For this reason we shall look more closely at the high temperature results from lattice gauge theory.

5   High Temperature Behavior of $\Delta(T)$

In this section we present a discussion of the properties of $\Delta(T)$ at temperatures above $2T_c$ up to around $5T_c$. Some of the early work in this temperature range was done using perturbative estimates by Källman [18] and Gorenstein and Mogilevsky [19]. Also Montvey and Pietarinen [20] looked at the asymptotic properties of the gluon gas. Källman suggested a mean-field type of model to which he fitted the lattice $SU(2)$ data [21] using a linear temperature dependence of the quantity $\Delta(T) \times T^4$ above T_c. At first appearance the data for $\Delta(T)$ fit rather well for the temperatures somewhat above the peak. A quite different analysis was carried out by Gorenstein and Mogilevsky, who compared the behavior of ϵ/T^4 and $3P/T^4$ as functions of $1/T^3$. Again it appeared that they approach each other at small values of $1/T^3$ using the best computer data available at the time [21]. With these new data for $SU(2)$ and $SU(3)$ [1,2] we are able to compare these various approaches to the high temperature behavior of $\Delta(T)$ at temperatures in the

above range. Figures 3(a) and 3(b) show the high temperature behavior of $SU(2)$ and $SU(3)$ respectively, as a function of $1/T^3$.

We see from these plots that the general appearance is much the same in the different cases. $\Delta(T)$ appears to be a linearly increasing function of the variable $1/T^3$. In Figure 3(b) we have provided the errors on the points so that we may assess the validity of our fitting procedure on the plots. We can see that for the $16^3 \times 4$ and the $32^3 \times 6$ both the data points and the curves are close with rather small errors, while for the $32^3 \times 8$ the highest point is well away from the interpolated curve, but still consistent within the error.

The question of how well these results can be compared to earlier work may now be answered. The assumption of glueballs of a mass m_{gb} gives a high temperature form for $\Delta(T)$ which is written as follows [20]:

$$\Delta(T) \;=\; \frac{d_{gb}}{6\pi^2}(m_{gb}/T)^3 K_1(m_{gb}/T), \tag{5.1}$$

where d_{gb} is the statistical degeneracy of the glueball states. In the high temperature limit for this case $\Delta(T)$ is a much more slowly rising function of $1/T^3$ than for the numerical results shown in the above plots. The asymptotic behavior of the form of $\Delta(T)$ assumed by Källman [18] and Gorenstein and Mogilevsky [19] is qualitatively similar to ours in the region where data exists. However, the latter [19] present their data in terms of energy
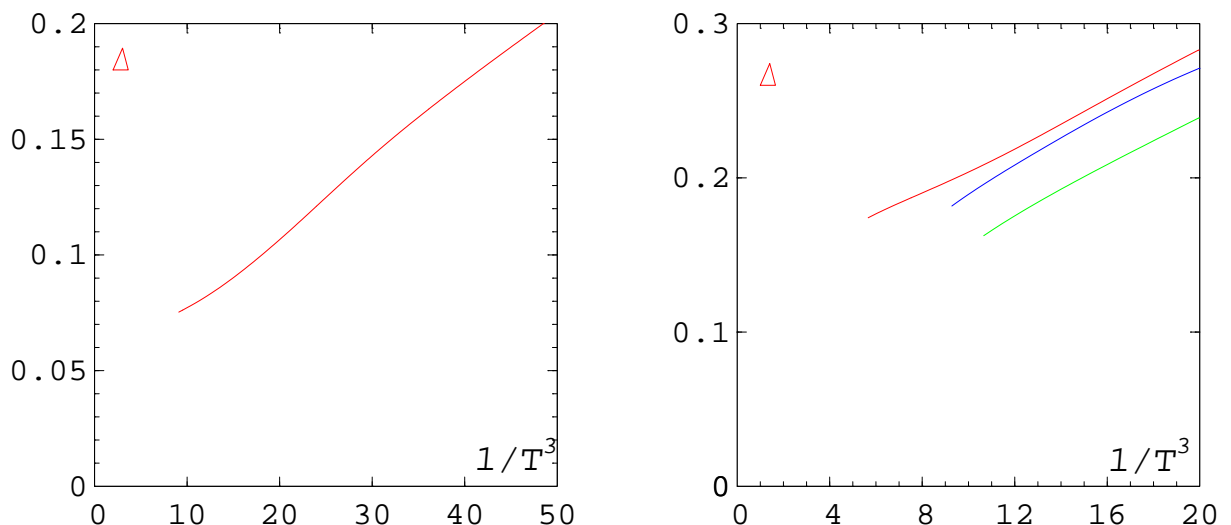

Figure 3: Figure (a) shows $\Delta(T)$ as a function of $1/T^3$ for $SU(2)$ lattice gauge theory [1],where the scale of the abscissa is $T_{\mathrm{c}}^{-3} \times 10^{-3}$; Figure (b) shows $\Delta(T)$ for $SU(3)$ with the three lattice sizes [2] $16^3 \times 4$ (top line), $32^3 \times 6$ (middle line), $32^3 \times 8$ (bottom line). The scale of the abscissa is the same as for $SU(2)$.

9

density and the pressure separately, and indicate that the two curves converge at small values of $1/T^3$. From our plot we see that this behavior is not supported by the present lattice data, rather there exists a considerable gap with no indication of $\Delta(T)$ approaching zero at small $1/T^3$.

If one were to consider a simple bag type of model with the bag constant B independent of the temperature, one finds immediately that $\Delta(T)$ is proportional to $1/T^4$. This dependence clearly results in a too rapid rise in $\Delta(T)$ as a function of $1/T^3$, which is not consistent with the data shown in Figure 3. plots. Although we are not able to exactly determine the asymptotic form of the function at high temperature, it appears to scale linearly with $1/T^3$ and to be non-zero at high temperature, ie., $\epsilon - 3p > 0$, for temperatures between $2T_c$ and $5T_c$.

6 Extension to full QCD

Finally we would like to discuss the changes due to the presence of dynamical quarks with a finite mass. There have been recent calculations of the thermodynamical quantities in full QCD with two flavours [22, 23], and with four flavours [24] of staggered quarks. These calculations are not yet as accurate as those in pure gauge theory for two reasons. The first is the prohibitive cost of obtaining statistics similar to those obtained for pure QCD. So the error on the interaction measure is considerably larger. The second reason, perhaps more serious, lies in the effect of the quark masses currently simulated. They are relatively heavy, which increases the contribution of the quark condensate term $m_q\langle\bar{\psi}\psi\rangle$ to the interaction measure.

The transition takes place when the temperature is high enough to excite many thermal mesons. So if the lightest mesons are heavier than in reality, due to the quark mass being heavier, the transition temperature should be higher. However, the lattice spacing is obtained by imposing the experimental rho mass, $m_\rho = 770\text{MeV}$, on the ρ for every quark mass simulated. This means that the temperature scale in physical units is itself not correct, and may well change considerably near T_c when smaller quark masses are simulated. The condensates in full QCD have also been considered by Koch and Brown [25], but the lattice measurements used were not fully non-perturbative, nor was the temperature scale obtained from the full non-perturbative beta-function.

In Figure 4 we have plotted the condensate from the $N_\tau = 6$ data of [23] for two flavors with $m_q/T = 0.075$, and preliminary results from Bielefeld [24] for four flavors with $m_q/T = 0.2$. The condensate is obtained from the interaction measure in a type of chiral limit; the simulation is performed with quarks of the stated mass, but the energy density is measured with the quark mass set to zero.

We have also indicated with a dashed line the pure gauge theory results of Figure 2 using $T_c = 0.150\text{GeV}$ to set the scale. The four flavor temperature scale has been converted
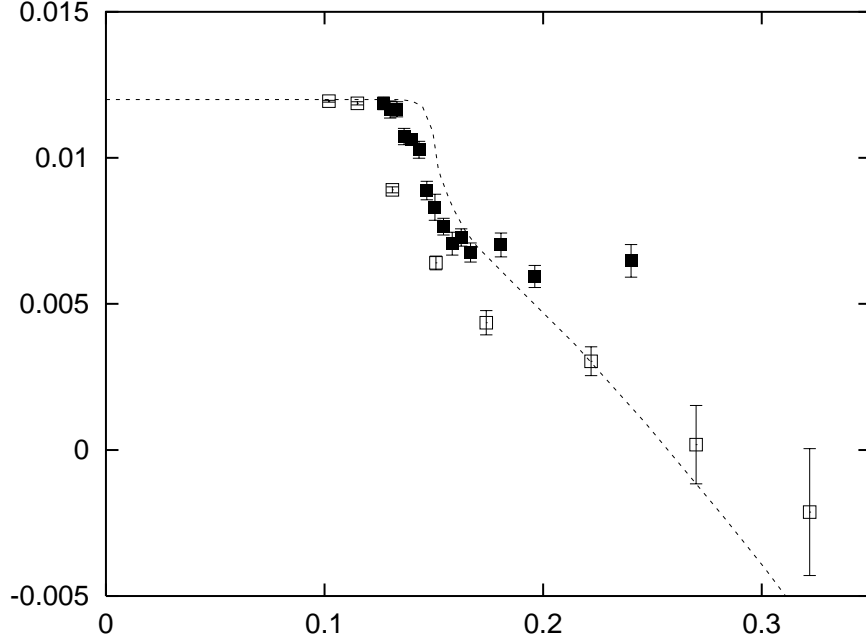
Figure 4: The gluon condensate in full QCD. The solid squares come from the two flavor [23], the open squares are for four flavous [24]. The dashed line shows the pure gauge condensate rescaled by the number of degrees of freedom. The units are the same as the previous figures.

to physical units using $T_{\mathrm{c}} = 0.130\mathrm{GeV}$ [26].

From this figure it is clear that no conclusion can yet be drawn for full QCD. The results of the MILC collaboration indicate that the condensate may flatten off above T_{c}. This behavior cannot be achieved by simpy rescaling the pure $SU(3)$ results, and suggests a qualitative change. If one assumes that the condensate should go to zero at high temperature, then this indicates that the zero temperature gluon condensate is not 0.012 GeV^4 but rather 0.007 GeV^4. Equivalently, one could claim that the condensate drops to half the zero temperature value in the high temperature phase.

Unfortunately the MILC data do not go much above T_{c}, so this conclusion depends very strongly on the last few points. The preliminary Bielefeld data, which go to higher temperatures, behave similarly to the rescaled pure $SU(3)$ data, and indicate that the condensate does become negative at high temperatures.

As mentioned above there are severe difficulties to be overcome when the temperature scale, coming from the full non-perturbative beta function, is fixed for full QCD simulations. It is quite plausible that the differences between the two simulations come from the different temperature scales used, and that more precise data will remove this difference. Also, the Bielefeld group uses an improved action for both the gluon and fermion parts

of the action, while the MILC group uses the standard action for both. So it is clearly too early to come to any conclusion about the high temperature behavior of the gluon condensate in full QCD.

Having said that, let us end by speculating on why it may be possible that full QCD behave differently from the pure gauge theory. A flattening off of the condensate suggests that the system is more stable because of the quarks. We consider the simple model presented in section 4, but now with the quarks included. The thermal quarks will change the interactions in the system due to the screening of the static interactions. Although there are still residual interactions in the magnetic sector even though the string tension itself goes to zero while the spatial string tension remains nonzero, it seems reasonable that the momentum dependent interactions that led to a negative condensate for pure gauge theory are screened at large distances in full QCD so that possibly no negative condensate forms at least within the range of the data.

7   Conclusions

The main conclusions for the temperature dependence of the gluon condensate come from the simulations of the $SU(N_\mathrm{c})$ lattice gauge theories at finite temperature [1, 2]. These simulations provide not only an accurate computation of the interaction measure $\Delta(T)$, but also of the temperature scale due to the calculation of the beta function at finite temperature. Both are vital in the computation of $\theta^\mu_\mu(T)$ in the previous sections. We have also given a discussion of these quantities in relation to full QCD.

It is clear that the condensate becomes negative in pure gauge theory, and keeps dropping with increasing temperature. The point at which it becomes zero has no special significance. This can be understood in terms of a model based on boson fields as coming from interactions between the thermal bosons.

For full QCD it is not yet possible to draw a conclusion. There are data supporting both the condensate going to zero and its becoming negative at high temperatures. Future simulations will show whether or not pure and full QCD have qualitatively different behaviours.

Although we are less able to say what happens to the temperature scale and the beta function in full QCD, we can see from the present newer computations as well as the known results [22] that $\theta^\mu_\mu(T)$ is always positive at all computed values of the temperature. Therefore, provided that the vacuum contribution to the trace of the energy momentum tensor is positive, we may conclude from Equation (1.1) that the total trace of $T^\mu_\mu(T)$ always remains positive. In this case the divergence of the dilatation and conformal currents remains positive. Thus these currents are not conserved. Therefore, the scale symmetry remains broken at all temperatures for nonabelian lattice gauge theories.

8   Acknowledgements

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